HOME WORK

**#1.4,1.5**

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**#1.4**

**10. Let C(x) be the statement “x has a cat,” let D(x) be the statement “x has a dog,” and let F(x) be the statement “x has a ferret.” Express each of these statements in terms of C(x), D(x), F(x), quantifiers, and logical connectives.**

**Let the domain consist of all students in your class.**

**a) A student in your class has a cat, a dog, and a ferret.**

**∃x (C(x) ∧ D(x) ∧ F(x))**

**b) All students in your class have a cat, a dog, or a ferret.**

**∀x (C(x) v D(x) v F(x))**

**c) Some student in your class has a cat and a ferret, but not a dog.**

**∃x (C(x) ∧ F(x) ∧￢D(x))**

**d) No student in your class has a cat, a dog, and a ferret.**

**￢∃x (C(x) ∧ D(x) ∧ F(x))**

**e) For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this animal as a pet.**

**(∃x C(x)) ∧ (∃x D(x)) ∧ (∃x F(x))**

**18. Suppose that the domain of the propositional function P(x) consists of the integer’s −2, −1, 0, 1, and 2. Write out each of these propositions using disjunctions, conjunctions, and negations.**

**a) ∃xP(x)**

**P (-2) v P (-1) v P (0) v P (1) v P (2)**

**b) ∀xP(x)**

**P (-2) ∧ P (-1) ∧ P (0) ∧ P (1) ∧ P (2)**

**c) ∃x￢P(x)**

**￢P (-2) v ￢P (-1) v ￢P (0) v ￢P (1) v ￢P (2)**

**d) ∀x￢P(x)**

**￢P (-2) ∧ ￢P (-1) ∧ ￢P (0) ∧ ￢P (1) ∧ ￢P (2)**

**e) ￢∃xP(x)**

**￢ (P (-2) v P (-1) v P (0) v P (1) v P (2))**

**f) ￢∀xP(x)**

**￢ (P (-2) ∧ P (-1) ∧ P (0) ∧ P (1) ∧ P (2))**

**30. Suppose the domain of the propositional function P(x, y) consists of pairs x and y, where x is 1, 2, or 3 and y is 1, 2, or 3. Write out these propositions using disjunctions and conjunctions.**

**a) ∃x P(x, 3)**

**P (1, 3) v P (2, 3) v P (3, 3)**

**b) ∀y P (1, y)**

**P (1, 1) ∧ P (1, 1) ∧ P (1, 3)**

**c) ∃y ￢P (2, y)**

**￢ P (2, 1) v ￢ P (2, 2) v ￢ P (2, 3)**

**d) ∀x ￢P(x, 2)**

**￢ P (1, 2) ∧￢ P (2, 2) ∧￢ P (3, 2)**

**40. Express each of these system specifications using predicates, quantifiers, and logical connectives.**

**a) When there is less than 30 megabytes free on the hard disk, a warning message is sent to all users.**

**∃x P(x) → ∀x Q(y)**

**b) No directories in the file system can be opened and no files can be closed when system errors have been detected.**

**∀x T (z) → [∀x (￢R(x)) ∧∀x (￢S(y))]**

**c) The file system cannot be backed up if there is a user currently logged on.**

**∃x ∃y (U(x, y) → ￢V(y))**

**d) Video on demand can be delivered when there are at least 8 megabytes of memory available and the connection speed is at least 56 kilobits per second.**

**∀x [W(x) ∧ A (x)) →￢B(x)]**

**54. Write out ∃!x P(x), where the domain consists of the integers 1, 2, and 3, in terms of negations, conjunctions, and disjunctions.**

**(P (1) v P (2) v P (3)) ∧￢(P(1)∧P(2))∧ ￢(P(2)∧P(3))∧ ￢(P(1)∧P(3))**

**60. Let P(x), Q(x), and R(x) be the statements “x is a clear explanation,” “x is satisfactory,” and “x is an excuse,” respectively. Suppose that the domain for x consists of all English text. Express each of these statements using quantifiers, logical connectives, and P(x), Q(x), and R(x).**

**a) All clear explanations are satisfactory.**

**∀x (P(x) →Q(x))**

**b) Some excuses are unsatisfactory.**

**∃x (R(x) ∧￢Q(x))**

**c) Some excuses are not clear explanations.**

**∃x (R(x) ∧￢P(x))**

**∗d) Does (c) follow from (a) and (b)?**

**Yes**

**#1.5**

**10. Let F(x, y) be the statement “x can fool y,” where the domain consists of all people in the world. Use quantifiers to express each of these statements.**

**a) Everybody can fool Fred.**

**b) Evelyn can fool everybody.**

**c) Everybody can fool somebody.**

**d) There is no one who can fool everybody.**

**e) Everyone can be fooled by somebody.**

**f ) No one can fool both Fred and Jerry.**

**g) Nancy can fool exactly two people.**

**h) There is exactly one person whom everybody can fool.**

**i) No one can fool himself or herself.**

**j) There is someone who can fool exactly one person besides himself or herself.**

**Answer:**

**a) ∀x F(x, Fred)**

**b) ∀y F (Evelyn, y)**

**c) ∀x ∃y F(x, y)**

**d) ￢∃x ∀y F(x, y)**

**e) ∀y ∃x F(x, y)**

**f) ￢∃x (F(x, Fred) ∧ F(x, Jerry))**

**g) ∃y1∃y 2(F (Nancy, y1) ∧ F (Nancy, y2) ∧ y1 \_= y2 ∧ ∀y(F(Nancy, y) → (y = y1 ∨ y = y2)))**

**h) ∃y (∀x F(x, y) ∧ ∀z (∀x F(x, z) → z = y)) i) ￢∃x F(x, x)**

**j) ∃x ∃y(x \_= y ∧ F(x, y) ∧∀z ((F(x, z) ∧ z \_= x) → z = y)) (We do not assume that this sentence is asserting that this person can or cannot fool her/himself.)**

**28. Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.**

**a) ∀x∃y(x2 = y)**

**b) ∀x∃y(x = y2)**

**c) ∃x∀y(xy = 0)**

**d) ∃x∃y(x + y \_= y + x)**

**e) ∀x(x \_= 0 → ∃y(xy = 1))**

**f ) ∃x∀y(y \_= 0 → xy = 1)**

**g) ∀x∃y(x + y = 1)**

**h) ∃x∃y(x + 2y = 2 ∧ 2x + 4y = 5)**

**i) ∀x∃y(x + y = 2 ∧ 2x − y = 1)**

**j) ∀x∀y∃z(z = (x + y)/2)**

**Answer:**

**a) True (let y = x 2 )**

**b) False (no such y exists if x is negative)**

**c) True (let x = 0)**

**d) False (the commutative law for addition always holds)**

**e) True (let y = 1/x)**

**f) False (the reciprocal of y depends on y —there is not one x that works for all y )**

**g) True (let y = 1− x)**

**h) False (this system of equations is inconsistent)**

**i) False (this system has only one solution; if x = 0, for example, then no y satisfies y = 2∧−y = 1)**

**j) True (let z = (x + y)/2)**

**34. Find a common domain for the variables x, y, and z for which the statement ∀x∀y((x \_= y) → ∀z((z = x) ∨ (z = y))) is true and another domain for which it is false.**

**True: {0, 1}**

**False: ℝ**

**46. Determine the truth value of the statement ∃x∀y(x ≤ y2) if the domain for the variables consists of**

**a) The positive real numbers.**

**b) The integers.**

**c) The nonzero real numbers.**

**Answer:**

**This statement says that there is a number that is less than or equal to all squares.**

**a) This is false, since no matter how small a positive number x we might choose, if we let y = \_x/2, then x = 2y2 , and it will not be true that x ≤ y2 .**

**b) This is true, since we can take x = −1, for example.**

**c) This is true, since we can take x = −1, for example.**

**A statement is in prenex normal form (PNF) if and only if it is of the form Q1x1Q2x2 · · ·QkxkP(x1, x2, . . . , xk), where each Qi, i = 1, 2, . . . , k, is either the existential quantifier or the universal quantifier, and P(x1, . . . , xk) is a predicate involving no quantifiers. For example, ∃x ∀y(P(x, y) ∧ Q(y)) is in prenex normal form, whereas ∃x P(x) ∨ ∀xQ(x) is not (because the quantifiers do not all occur first). Every statement formed from propositional variables, predicates, T, and F using logical connectives and quantifiers is equivalent to a statement in prenex normal form.**

**50. Put these statements in prenex normal form[Hint: Use logical equivalence from Tables 6 and 7 in Section 1.3,Table 2 in Section 1.4, Example 19 in Section 1.4, Exercises 45 and 46 in Section 1.4, and Exercises 48 and 49.]**

**a) ∃xP(x) ∨ ∃xQ(x) ∨ A, where A is a proposition not involving any quantifiers.**

**b) ￢ (∀xP(x) ∨ ∀xQ(x))**

**c) ∃xP(x) → ∃xQ(x)**

**Answer:**

**a) By Exercises 45 and 46b in Section 1.3, we can simply bring the existential quantifier outside: ∃x (P(x) ∨ Q(x) ∨ A).**

**b) By Exercise 48 of the current section, the expression inside the parentheses is logically equivalent to ∀x ∀y (P(x) ∨ Q(y)). Applying the negation operation, we obtain ∃x ∃y￢ (P(x) ∨ Q(y)).**

**c) First we rewrite this using Table 7 in Section 1.2 as ∃x Q(x) ∨ ￢∃x P(x), which is equivalent to ∃x Q(x) ∨∀x￢ P(x). To combine the existential and universal statements we use Exercise 49b of the current section, obtaining ∀x ∃y (￢P(x) ∨ Q(y)), which is in prenex normal form.**